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ABSTRACT BOOK

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Contents

Characterization of Riemannian manifolds with certain vector fields

Sudhakar Kumar Chaubey[1]

[1]Section of Mathematics, Department of Information Technology, University of Technology and Applied Sciences, Shinas, P.O. Box 77 Postal Code 324, Oman

Corresponding Author's E-mail: sudhakar.chaubey@shct.edu.om

Abstract

A new class of Riemannian manifold with certain vector field, named as Riemannian concircular structure manifold, is defined. Some basic curvature identities and integrability condition of such manifolds are established. The existence of Riemannian concircular structure manifolds are ensured by few non-trivial examples.

***K*eywords** Riemannian manifolds · Torse forming vector field · Symmetric spaces · Riemannian concircular structure manifolds · Quasi-Einstein manifolds

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On some generalized Einstein metric conditions

Ryszard Deszcz[1], Małgorzata Głogowska[2]

[1]Retired Employee of the Department of Applied Mathematics, Wrocław University of Environmental and Life Sciences, Grunwaldzka 53, 50-357 Wrocław, Poland

[2]Department of Applied Mathematics, Wrocław University of Environmental and Life Sciences, Grunwaldzka 53, 50-357 Wrocław, Poland

Corresponding Author's E-mail: Ryszard.Deszcz@upwr.edu.pl

Abstract

Let (M, g) be a semi-Riemannian manifold. We denote by g, R, S, κ and C the metric tensor, the Riemann-Christoffel curvature tensor, the Ricci tensor, the scalar curvature and the Weyl conformal curvature tensor of (M, g), respectively. Using these tensors we can define the $(0, 6)$-tensors $R \cdot R$, $R \cdot C$, $C \cdot R$, $C \cdot C$ and $Q(A, T)$, where A is a symmetric $(0, 2)$-tensor and T a generalized curvature tensor (see, e.g., [9], [11] and [14]).

A semi-Riemannian manifold (M, g), $\dim M = n \geq 2$, is said to be an *Einstein manifold* [2], or an *Einstein space*, if at every point of M its Ricci tensor S is proportional to g, i.e., $S = (\kappa/n)\, g$ on M, assuming that κ is constant when $n = 2$. According to [2, p. 432], this condition is called the *Einstein metric condition*. Einstein manifolds form a natural subclass of several classes of semi-Riemannian manifolds which are determined by curvature conditions imposed on their Ricci tensor [2, Table, pp. 432-433]. These conditions are called *generalized Einstein metric conditions* [2, Chapter XVI].

The tensor $R \cdot C - C \cdot R$ of every semi-Riemannian Einstein manifold (M, g), $n \geq 4$, satisfies the following identities [12, Theorem 3.1] (see also [8, p. 100001-1] and [14, p. 107])

$$R \cdot C - C \cdot R = \frac{\kappa}{(n-1)n}\, Q(g, R) = \frac{\kappa}{(n-1)n}\, Q(g, C) = \frac{1}{n-1}\, Q(S, R) = \frac{1}{n-1}\, Q(S, C),$$

$$R \cdot C - C \cdot R = \frac{\kappa}{n-1}\, Q(g, C) - Q(S, C) = Q(\frac{\kappa}{n-1}\, g - S, C). \qquad (1)$$

We can express the tensor $R \cdot C - C \cdot R$ of some non-Einstein and non-conformally flat semi-Riemannian manifolds (M, g), $\dim M \geq 4$, as a linear combination of $(0, 6)$-Tachibana tensors $Q(A, T)$, e.g., $A = g$ or $A = S$ and $T = R$ or $T = C$. These conditions form a family of generalized Einstein metric conditions. Semi-Riemannian manifolds, and in particular hypersurfaces isometrically immersed in spaces of constant curvature, satisfying such conditions were investigated in several papers, see, e.g., [1, 4, 7, 9, 10, 11, 13, 14]. We refer to [5] (see also [8]) for a survey of results on manifolds (hypersurfaces) satisfying such conditions.

Below we present selected results related to our talk.

If a non-quasi-Einstein and non-conformally flat semi-Riemannian manifold (M, g), $n \geq 4$, satisfies the following two curvature conditions of pseudosymmetry type: $R \cdot R = L_1\, Q(g, R)$ and $C \cdot C = L_2\, Q(g, C)$, where L_1 and L_2 are some functions, then the curvature tensor R is a linear combination of the Kulkarni-Nomizu products formed by the metric tensor g and the Ricci tensor S [15, Theorem 3.1, Teorem 3.2 (ii)]. A non-quasi-Einstein and non-conformally flat semi-Riemannian manifold (M, g), $n \geq 4$, with curvature tensor R expressed by the above-mentioned linear combination of the Kulkarni-Nomizu products is called a *Roter type manifold*, or a *Roter manifold*, or a *Roter space* (see, e.g., [3, Section 15.5], [5, 9, 11], [14, Section 4]). Every Roter space satisfies (1) (see, e.g., [9, Proposition 3.3], [11, Theorem 2.4 (ii)]).

Let M, $\dim M \geq 4$, be a hypersurface isometrically immersed in a space of constant curvature such that at every point M has exactly two distinct principal curatures, λ_1 with multiplicity p and λ_2 with multiplicity $n - p$, $2 \leq p \leq n - 2$. If $(p_1 - 1)\lambda_1 + (n - p - 1)\lambda_2 \neq 0$, then M is a Roter space, and in a consequence (1) holds on M [6, Theorem 3.3].

Let M, $\dim M \geq 4$, be a hypersurface isometrically immersed in a space of constant curvature such that at every point M has exactly three distinct principal curatures. If the condition $R \cdot C - C \cdot R = Q(g, T)$, where T is a generalized curvature tensor, is satisfied on M, then the tensor T is a linear combination of the curvature tensor R and Kulkarni-Nomizu products formed by the metric tensor g, the Ricci tensor S and its square S^2 (cf. [10, Theorem 5.2]).

If at every point of a non-quasi-Einstein and non-conformally flat hypersurface M, $\dim M \geq 4$, isometrically immersed in a semi-Riemannian space of constant curvature the tensor $R \cdot C - C \cdot R$ is a linear combination of the tensors $Q(g, C)$ and $Q(S, C)$, then (1) holds on M [11, Theorem 5.4].

Keywords Einstein manifold · Einstein generalized metric condition · pseudosymmetric type curvature condition · Roter space · hypersurface

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CAUCHY-RIEMANN GEOMETRY : AN INTRODUCTION TO THE MAIN PROBLEMS

Sorin Dragomir[1]

[1] Universit'a degli Studi della Basilicata, Dipartimento di Matematica, Informatica ed Economia, Potenza, Italy

Corresponding Author's E-mail: sorin.dragomir@unibas.it

ABSTRACT

The ordinary Cauchy-Riemann system $\bar{\partial} f = 0$ on $\mathbb{C}^n$ ($n \geq 2$) induces on every real hypersurface $M \subset \mathbb{C}^n$ the tangential Cauchy-Riemann equations

$$\bar{\partial}_b u = 0 \tag{2}$$

(a first order overdetermined PDE system, with variable C^∞ coefficients) and every C^1 solution $u : M \to \mathbb{C}$ to (1) is a CR function on M. A CR structure is a recast of (1) as an involutive complex distribution $T_{1,0}(M) \subset T(M) \otimes \mathbb{C}$, of complex rank n-1, and the restriction to M of a holomorphic function (on a neighborhood $\Omega \supset M$) is a solution to (1). The CR extension problem is whether a point $x_0 \in M$ admits a neighborhood $\Omega \subset \mathbb{C}^n$ such that the restriction morphism $\mathcal{O}(\Omega) \to CR^1(U)$ is an epimorphism (with $U = \Omega \cap M$). Given an abstract CR structure $T_{1,0}(M)$ on a real $(2n-1)$-dimensional manifold (not necessarily embedded into $\mathbb{C}^n$) the CR embedding problem is whether a point $x_0 \in M$ admits a neighborhood $U \subset M$ and a CR immersion $\Psi : U \to \mathbb{C}^n$ [so that the portion of $T_{1,0}(M)$] over U is actually induced by the complex structure of the ambient space $\mathbb{C}^n$]. We review results (old and new) on the two fundamental problems mentioned above, with an emphasis on the differential geometric objects needed in their study (cf. [3] and [1]), and indicate their relationship to mathematical physics (cf. [7], [8], [5], and [6]).

Keywords Tangential CR equations · CR function · Tanaka-Webster connection · sublaplacian · Fefferman metric · Robinson-Trautman construction

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PMC Biconservative Surfaces in Complex Space Forms

Prof. Dorel FETCU[1]

[1]Asachi Technical University of Iasi, Romania

Corresponding Author's E-mail: etti.tuiasi.ro

Abstract

We consider PMC surfaces in complex space forms, and discuss about the interaction between the notions of PMC, totally real and biconservative. We will see that PMC surfaces in a non-flat complex space form are biconservative if and only if totally real. Then, we will present a Simons type formula for a well-chosen vector field constructed from the mean curvature vector field and then a rigidity result for CMC biconservative surfaces in 2-dimensional complex space forms. We will also present a reduction of codimension result for PMC biconservative surfaces in non-flat complex space forms. We conclude by constructing examples of CMC non-PMC biconservative submanifolds from the Segre embedding, and discuss when they are proper-biharmonic.

***K*eywords** complex space form · biharmonic surfaces · biconservative surfaces · PMC surfaces · CMC submanifolds · Segre embedding

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On the Divergence of the Weyl Tensor of Homogeneous Four-Manifolds

Eduardo GARCIA-RIO[1]

[1]*C*ITMAga, Faculty of Mathematics, University of Santiago de Compostela, 15782 Santiago de Compostela, Spain

Corresponding Author's E-mail: eduardo.garcia.rio@usc.es

Abstract

Four-dimensional homogeneous manifolds whose self-dual Weyl conformal curvature tensor is divergence free are classified. It is shown that, besides the symmetric case, only two geometries may appear. Anti-self-dual homogeneous four-manifolds (as described by de Smedt and Salamon) provide the first class, which is locally coformally equivalent to a Ricci flat Kahler surface. The other class corresponds to the only 3-symmetric manifold in dimension four, which is an algebraic Bach soliton. During the lecture we summarize the basic arguments in the proof and pay special attention to the geometry of the solution spaces.

***K*eywords** Homogeneous manifolds · Weyl conformal · divergence

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GEOMETRIC STRUCTURES ON THE FRAME BUNDLE OF A DIFFERENTIABLE MANIFOLD

Mohammad Nazrul Islam Khan[1]

[1]*D*epartment of Computer Engineering, College of Computer, Qassim University, Saudi Arabia

Corresponding Author's E-mail: m.nazrul@qu.edu.sa

ABSTRACT

We study the geometric structures such as metallic structures, almost r-contact structure on the frame bundle FM. A tensor field $\tilde{J}$ is defined on FM and shows that it is a metallic structure on FM. Let g^D be the diagonal lift of a Riemannian metric g. We investigate the diagonal lift g^D is a metallic Riemannian metric on FM. We discuss some results on the derivative and coderivative of 2-form F of metallic Riemannian structure on FM. Next, the Nijenhuis tensor of tensor field $\tilde{J}$ on FM is calculated. Finally, a locally metallic Riemannian manifold (FM, J^H, g^D) is described as an application.

***K*eywords** Metallic structure · Frame bundle · Diagonal lift · Nijenhuis tensor · 2-Form · Derivative

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SURFACES ASSOCIATED WITH PASCAL AND CATALAN TRIANGLES

Adela MIHAI[1]

[1]Technical University of Civil Engineering Bucharest, Romania and Transilvania University of Brasov, Romania

Corresponding Author's E-mail: adela.mihai@utcb.ro

ABSTRACT

An open problem in reliability theory is that of finding all the coefficients of the reliability polynomial associated with particular networks. Because reliability polynomials can be expressed in Bernstein form (hence linked to binomial coefficients), it is clear that an extension of the classical discrete Pascal's triangle (comprising all the binomial coefficients) to a continuous version (exhibiting infinitely many values in between the binomial coefficients) might be geometrically helpful and revealing. We investigated some geometric properties of a continuous extension of Pascal's triangle: Gauss curvature, mean curvatures, geodesics and level curves, as well as their symmetries. These results have been published in [1]. Also, in [2] we investigated surfaces associated with Catalan triangles.

***K*eywords** Reliability theory · Catalan triangle · Pascal triangle

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Manifolds with Special Holonomy and Applications

Sema Salur[1]

[1] *U*niversity of Rochester, Department of Mathematics, Rochester, NY, USA

Corresponding Author's E-mail: sema.salur@rochester.edu

Abstract

During the past few decades, M-theory, a "theory of everything", has become a very active and exciting area of research as a leading candidate to unify the four fundamental forces of nature–electromagnetism, gravity, the weak and strong nuclear forces.
In this talk we will discuss manifolds with special holonomy, spaces whose infinitesimal symmetries play an important role in M-theory compactifications. We will first give brief introductions to Calabi-Yau and G2 manifolds and then a survey of my recent research on relations between symplectic/contact/calibrated structures on manifolds with special holonomy.

***K*eywords** Manifolds with Special holonomy · Calibrations · Contact Structures · Symplectic Structures

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Ricci Solitons and Certain Related Metrics on Almost Co-Kähler Manifolds

Venkatesha Venkatesha[1]

[1] Karnata University, India

Corresponding Author's E-mail: vensmath@gmail.com

Abstract

We study a Ricci soliton and a generalized m-quasi-Einstein metric on almost co-Kaehler manifold M satisfying a nullity condition. First, we consider a non-co-Kaehler (κ, μ)-almost co-Kaehler metric as a Ricci soliton and prove that the soliton is expanding with $\lambda = -2n\kappa$ and the soliton vector field X leaves the structure tensors η, ξ and φ invariant. This result extends Theorem 5.1 of Suh and De. We construct an example to show the existence of a Ricci soliton on M. Finally, we prove that if M is a generalized (κ, μ)-almost co-Kaehler manifold of dimension higher than 3 such that $h \neq 0$, then the metric of M can not be a generalized m-quasi-Einstein metric, and this recovers the recent result of Wang as a special case.

***K*eywords** Almost co-Kähler manifold · Ricci soliton · Generalized m-quasi-Einstein metric · (κ, μ)-nullity distribution

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Higher Power Harmonic Maps and Sections

Chris Wood[1]

[1] Department of Mathematics, University of York, Heslington, York YO10 5DD, UK

Corresponding Author's E-mail: chris.wood@york.ac.uk

Abstract

I will give a brief overview of the variational theory of the family of higher-power energy functionals for mappings between Riemannian manifolds, and its generalisation to sections of submersions of Riemannian manifolds. I will then show how this may be applied to sections of Riemannian vector bundles and their sphere subbundles. The main example will be Ramachandran's complete classification of all higher-power harmonic left-invariant vector fields on 3-dimensional unimodular Lie groups equipped with an arbitrary left-invariant Riemannian metric. This rounds out the picture for harmonic vector fields in these cases, obtained by Gonzalez-Davila and Vanhecke.

Keywords Riemannian manifolds · Elementary invariants · Newton polynomials · Higher-power energy and vertical energy · Newton tensors · 3-dimensional unimodular Lie groups

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MODULI SPACE OF SINGULAR MODELS OF $K3$-SURFACES

Çisem GÜNEŞ AKTAŞ

*A*bdullah Gül University, Türkiye

Corresponding Author's E-mail: cisem.gunesaktas@agu.edu.tr

ABSTRACT

A projective model $X \to \mathbb{P}^{(n+1)}$ of a $K3$-surface X gives rise to a polarization, i.e., class $h \in H_2(X)$ of the hyperplane section, so that $h^2 = 2n$. Then, irreducible smooth rational curves on X are (some) classes $l \in NS(X) \subset H_2(X)$ of square (-2) intersecting h in a prescribed way. The simplest case is that of exceptional divisors, $l \cdot h = 0$, in which we are speaking about the equisingular deformation classification of singular models. The "polarization" $h^2 = 0$ (elliptic $K3$-surfaces) was recently settled by Shimada [7]. The case of plane sextic curves where $h^2 = 2$ was closed in Akyol and Degtyarev [5], after numerous and almost fruitless attempts to settle it by the conventional, equation based methods (see, e.g., Artal et al. [1, 2, 3] or Oka and Pho [4]). The next case of spatial quartics where $h^2 = 4$ was treated conventionally by Degtyarev [6] in the presence of a non-simple singularity, i.e., when the quartic is not a K3-surface; already then it became apparent that extending these methods to surfaces with simple singularities only is hopeless. In this research we use the K3-theoretic approach which is also pioneered by Urabe [8, 9] and Yang [10], to give a complete description of the moduli space of quartic surfaces which are surfaces in $\mathbb{P}^3$ of degree 4. We confine ourselves to *simple* quartics only, *i.e.*, those with **A–D–E** type singularities. Two such quartics are said to be *equisingular deformation equivalent* if they belong to the same deformation family in which the total Milnor number stays constant. Four seems to be the last degree where one can hope to obtain a complete equisingular deformation classification. *Simple* quartics are $K3$-surfaces, and as such they can be studied by using the global Torelli theorem and the surjectivity of the period map, combined with Nikulin's theory of discriminant forms. This approach was used by Urabe [9, 8], who showed that the total Milnor number μ of a simple quartic does not exceed 19 and listed (in terms of perturbations of Dynkin graphs) all realizable sets of singularities with the total Milnor number $\mu \leq 17$. In this talk, we present a systematic study of the moduli space of simple quartic surfaces confining ourselves to the so-called *non-special* quartics.

***K*eywords** $K3$-surface · moduli space · singular model · simple quartic

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Weakly Symmetric Kenmotsu Manifolds with a Non-Symmetrics Non-Metric Connection

Selahattin Beyendi

*I*nonu University, Department of Mathematics, Faculty of Education, 44280, Malatya, Türkiye

selahattin.beyendi@inonu.edu.tr

Abstract

Among the most outstanding topics in the field of differential geometry in recent years is the contact geometry. The geometry of contact Riemmanian manifolds and related issues have also received great attention in recent years. An important class of contact manifolds is the Kenmotsu manifolds. The notion of Kenmotsu manifolds was defined by K. Kenmotsu [1]. Kenmotsu manifolds have been extensively studied from various perspectives. In these studies, linear connection are among the great attention studies. In this respect, the idea of semi-symmetric linear connection in Riemannian manifolds was first put forward by Friedmann and Schouten [5] in 1924. Hayden [6] in 1932, introduced and studied the idea of semi-symmetric linear connection with torsion on a Riemannian manifold. After a long interval, Yano [15] started the systematic study of a semi-symmetric metric connection on a Riemannian manifold in 1970. A linear connection ∇ in (M, g), whose torsion tensor T of type $(1, 2)$ is defined as

$$T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y]$$

for arbitrary vector fields X and Y. The connection ∇ is symmetric if the torsion tensor T vanishes, otherwise it is non-symmetric. A semi-symmetric connection ∇ is said to be a semi-symmetric metric connection if $\nabla g = 0$, otherwise it is said to be a semi-symmetric non-metric connection. The semi-symmetric non-metric connection in a Riemannian manifold have been studied by [1, 2, 3] and many others. Recently, non-symmetric non-metric connection have been studied by [8, 9, 12].

On the other hand, in 1989, weakly symmetric and weakly Ricci symmetric manifolds were first studied by Tamassy and Binh [14]. De and Ghosh defined the weakly concircular Ricci symmetric manifolds [4]. Shaikh and Hui initiated the notion of weakly concircular symmetric manifolds [13]. In 2015, Prakasha and Vikas

studied weakly symmetric properties of Kenmotsu manifolds admitting a quarter symmetric metric connection [10]. In 2019, Singh and Lalnunsiami sudied some results on weakly symmetric Kenmotsu manifolds [11]. Motivated by these studies, we study weakly symmetric Kenmotsu manifolds with a non-symmetric non-metric connection.

***K*eywords** Kenmotsu manifolds, weakly symmetric, non-symmetric non-metric connection.

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Existence Solutions of a Quasilinear Wave Problem with Strong Damping

Abir BOUNAAMA[1]

[1]University of 20 August 1955 (LAMAHIS) Skikda, Algeria

Corresponding Author's E-mail: a.bounaama@univ-skikda.dz

Abstract

In this paper, firstly we show the local existence theorem for wave hyperbolic equation and we study the global existance of solutions of the wave equation . This article is organized as follows: In the first section : we start by introduction. In the second section : we show the local existence theorem. In the third section , we prove the results of the global existence of solutions.

Keywords global existence · source terms · weak solutions · hyperbolic equation

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Existence results for discrete fractional with three-point boundary value problems

Omar Choucha[1], Abdelkader Amara[2], Sina. Etemad[3]

[1]Laboratory of Applied Mathematics, University of Kasdi Merbah Ouargla 30000, Algeria
[2]Laboratory of Applied Mathematics, University of Kasdi Merbah Ouargla 30000, Algeria
[3]Department of Mathematics, Azarbaijan Shahid Madani University, Tabriz, Iran

Corresponding Author's E-mail: choucha.omar@univ-ouargla.dz,amara.abdelkader@univ-ouargla.dz 2,sina.etemad@gmail.com

Abstract

We prove existence and uniqueness of solutions to discrete fractional equations that involve Riemann-Liouville and Caputo fractional derivatives with three-point boundary conditions. The results are obtained by conducting an analysis via the Banach principle and the Brouwer fixed point criterion. Finally, some numerical models are provided to illustrate and validate the theoretical results.

Fractional difference calculus is a tool used to explain many phenomena in physics, control problems, modeling, chaotic dynamical systems, and various fields of engineering and applied mathematics. In this direction, different kinds of methods and techniques, including numerical and analytical methods, have been utilized by researchers to discuss given fractional discrete and continuous mathematical models and boundary value problems (BVPs) . For some recent developments on the existence, uniqueness, and stability of solutions for fractional differential equations, see, for example, [5, 1, 4, 2, 3] and the references therein.

Discrete fractional calculus and difference equations open a new study context for mathematicians. For this reason, they have received increasing attention in recent years. Some real-world processes and phenomena are analyzed with the aid of discrete fractional operators, since such operators provide an accurate tool to describe memory. A large number of research articles dealing with difference equations and discrete fractional boundary value problems (FBVPs) can be found in [5, 3, 2].

In 2020, Selvam et al [1] proved the existence of a solution to a discrete fractional difference equation formulated as

$$\begin{cases} {}^{c}\Delta_{\xi}^{\varrho}\chi(\xi) = \Phi(\xi+\varrho-1, \chi(\xi+\varrho-1)), \quad 1<\varrho\leq 2, \\ \Delta\chi(\varrho-2) = M_1, \qquad \chi(\varrho+T) = M_2, \end{cases} \tag{3}$$

for $\xi \in [0,T]_{\mathbb{N}_0} = [0,1,2,\ldots,T]$, $T \in \mathbb{N}$, $\eta \in [\varrho-1, T+\varrho-1]_{\mathbb{N}_{\varrho-1}}$, M_1 and M_2 constants, $\Phi : [\varrho-2, \varrho+T]_{\mathbb{N}_{\varrho-2}} \times \mathbb{R} \longrightarrow \mathbb{R}$ continuous, and where ${}^c\Delta_\xi^\varrho$ denotes the ϱth-Caputo difference. Here, motivated by the discrete model (3), we shall consider two generalized discrete problems. Our goal consists to study existence and uniqueness of solutions to the following discrete fractional equation that involves Caputo discrete derivatives:

$$\begin{cases} {}^c\Delta_\xi^\varrho \chi(\xi) = \Phi(\xi+\varrho-1, \chi(\xi+\varrho-1)), \quad 2 < \varrho \leq 3, \\ \Delta\chi(\varrho-3) = A_1, \chi(\varrho+T) = \lambda\Delta^{-\beta}\chi(\eta+\beta), \Delta^2\chi(\varrho-3) = A_2, \end{cases} \tag{4}$$

for $0 < \beta \leq 1$, $\xi \in [0,T]_{\mathbb{N}_0} = [0,1,2,\ldots,T]$, $T \in \mathbb{N}$, $\eta \in [\varrho-1, T+\varrho-1]_{\mathbb{N}_{\varrho-1}}$, λ, A_1 and A_2 constants, and where $\Phi : [\varrho-3, \varrho+T]_{\mathbb{N}_{\varrho-3}} \times \mathbb{R} \longrightarrow \mathbb{R}$ is continuous.

Keywords discrete fractional operators · , stability · , existence results · , Banach principle

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On optimal inequalities involving δ-Casorati curvature

Majid Ali Choudhary[1]

[1]Department of Mathematics, School of Sciences, Maulana Azad National Urdu University, Hyderabad, India

Corresponding Author's E-mail: majid_alichoudhary@yahoo.co.in

Abstract

Chen [2] constructed a sharp inequality to establish Chen's invariants, also known as δ-invariants, in order to investigate the link between intrinsic and extrinsic invariants. This discovery enabled the development of a new branch of differential geometry, and the study of Chen invariants and Chen-type inequalities for various submanifolds in various ambient spaces were conducted. It was possible to define the optimal inequality for submanifolds in various ambient spaces by substituting the Casorati curvature for the traditional Gauss curvature in [1]. The study of δ-Casorati curvatures in the context of δ-invariants was initiated by Decu et al [3]. This presentation provides a comprehensive examination of recent work on δ-Casorati curvatures from the past several years.

Keywords Optimal inequality · δ-invariant · Slant submanifolds · Chen invariant · Ideal submanifold · δ-Casorati curvatures

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DIFFERENTIAL GEOMETRY OF UMBRELLA MATRICES

Yusuf YAYLI[1], Mert ÇARBOĞA[2]

[1,2]Department of Mathematics, Faculty of Science, Ankara University, Türkiye

Corresponding Author's E-mail: mcarboga@ankara.edu.tr

ABSTRACT

In this paper, studies on umbrella matrices will be discussed. Erdoğan Esin's work with umbrella matrices will be mentioned.

Keywords Umbrella matrices · Differential geometry

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ON SOME PROPERTIES OF RICCI SOLITONS

Yusuf DOĞRU[1],

[1]*H*ava Eğitim Komutanlığı, Türkiye

Corresponding Author's E-mail: yusufdogru1@yahoo.com

ABSTRACT

Let (M, g) be a Riemannian manifold. R. S. Hamilton was introduced the *Ricci flow* $\frac{\partial}{\partial t}g(t) = -2Ric(g(t))$.The Ricci flow is an evolution equation for Riemannian metrics. Ricci solitons correspond to self-similar solutions of Ricci flow. A smooth vector field υ on a Riemannian manifold (M, g) is considered to define a *Ricci soliton* should a real constant exist λ such that $\frac{1}{2}\pounds_\upsilon g + Ric = \lambda g$,where $\pounds_\upsilon$ denotes the Lie derivative operator in the direction of the vector field υ, Ric denotes the Ricci curvature. A vector field υ on a Riemannian manifold (M, g) is called recurrent if

$\nabla_{X_1}\upsilon = \varphi(X_1)\upsilon$, where φ is a 1-form and ∇ is the Levi-Civita connection of g. we consider some properties of Ricci solitons on Riemannian manifolds and their submanifolds when the potential vector field is a recurrent vector field. Under some certain conditions, we show that the manifold (or submanifold) is a generalization of an Einstein manifold. Let (M, g) be a Riemannian manifold admitting an affine

connection, U a parallel unit vector field with respect to the Levi-Civita connection ∇ and υ a recurrent vector field with respect non-metric affine connection on M. Assume that a 1-form ϕ is the g-dual of υ. Then (M, g) is a Ricci soliton (υ, λ) if and only if M is a *hyper-generalized quasi-Einstein manifold.* Let M be a $\upsilon^\perp$-umbilical submanifold isometrically immersed into a Riemannian manifold $\left(\widetilde{M}, \widetilde{g}\right)$ endowed with a a non-metric affine connection and υ a recurrent vector field with respect to a non-metric affine connection on $\widetilde{M}$. Assume that a 1-form ϕ is the g dual of υ^T. Then (M, g) is a Ricci soliton (υ^T, λ) if and only if it is a hyper-generalized quasi-Einstein manifold.

***K*eywords** Ricci soliton, Riemannian manifolds, potential vector field, recurrent vector field.

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INEXTENSIBLE FLOWS OF FRAMED CURVE IN 4-DIMENSIONAL EUCLIDEAN SPACE

Hülya Gün Bozok[1], Onur Talip Dostuoğlu[2]

[1]Department of Mathematics,Faculty of Arts and Sciences, Osmaniye Korkut Ata University, Osmaniye, Türkiye
[2]Department of Mathematics,Institute of Natural and Applied Sciences, Osmaniye Korkut Ata University, Osmaniye, Türkiye

Corresponding Author's E-mail: hulyagun@osmaniye.edu.tr

ABSTRACT

The flow of a curve is said to be inextensible if its arclength is preserved. Physically, the inextensible curve flows give rise to motions in which no strain energy is induced. Such motions arise quite naturally in a wide range of the physical applications. The flows of inextensible curve and surface are one of the tool to solve many problems in computer vision, computer animation and even structural mechanics. In recent years, numerous studies are done about inextensible flows of curves by many mathematicians in different space.

On the other hand in Euclidean space there is possible to define the Frenet equations and curvatures of a regular curve. However, a space curve may have some singular points in Euclidean space. In singular points the Frenet frame can not be defined. Honda and Takahashi gave the definition of a framed curve to study curves with these points[2]. A framed curve is a space curve with a moving frame and it is a generalization of Frenet curve. Consequently with this new definition a lot of studies are done. Such that framed rectifying curve is introduced and the properties of framed rectifying curves are researched in [4]. Framed normal curves are investigated in [1]. Also evolution of framed curves in 3-dimensional Euclidean space is obtained in [3].

In this paper we study inextensible flows of framed curves in 4-dimensional Euclidean space . We obtain evolution equations of framed curve. Also we give necessary and sufficient conditions for inextensible flows of framed curves which are expressed as a partial differential equation involving the framed curvature in 4-dimensional Euclidean space.

Keywords inextensible flow · evolution equation · framed curve

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THREE CLASSIFICATION THEOREMS FOR TOTALLY UMBILICAL SEMI-INVARIANT SUBMANIFOLDS OF LOCALLY DECOMPOSABLE METALLIC RIEMANNIAN MANIFOLDS

Mustafa GÖK

Sivas Cumhuriyet University, Department of Design, Sivas, Türkiye

mgok@cumhuriyet.edu.tr; mustafa.gok@email.com

ABSTRACT

The study of submanifolds of metallic Riemannian manifolds was initiated by C. E. Hreţcanu and M. C. Crâşmăreanu in [5]. Some different kinds of submanifolds in metallic Riemannian manifolds, such as invariant, anti-invariant, slant, semi-slant, hemi-slant, bi-slant submanifolds were introduced and analyzed their fundamental properties by C. E. Hreţcanu and A. M. Blaga from the point of the characterization, the integrability of the associated distributions, totally mixed geodesicity and the parallelism of induced canonical structures (see, e.g., [1, 2, 3, 4] and the reference therein).

The concept of a semi-invariant submanifold in metallic Riemannian manifolds was defined by C. E. Hreţcanu and A. M. Blaga in [3] as a particular case of bi-slant submanifolds. An isometrically immersed submanifold M of a metallic Riemannian manifold $(\overline{M}, \overline{g}, \overline{J})$ is said to be semi-invariant if there exists a pair of orthogonal complementary distributions D and $D^{\perp}$ on M such that $TM = D \oplus D^{\perp}$, $\overline{J}D = D$ and $\overline{J}D^{\perp} \subseteq TM^{\perp}$, where D and $D^{\perp}$ are called invariant and anti-invariant distributions, respectively. Moreover, if neither $D = \{0\}$ nor $D^{\perp} = \{0\}$, then M is named a proper semi-invariant submanifold.

In this talk, we discuss the classification theorems on a special type of semi-invariant submanifolds in locally decomposable metallic Riemannian manifolds, namely totally umbilical semi-invariant submanifolds. We consider an arbitrary totally umbilical proper semi-invariant submanifold M of a locally decomposable metallic Riemannian manifold $(\overline{M}, \overline{g}, \overline{J})$ with $TM = D \oplus D^{\perp}$, where D and $D^{\perp}$ are the distributions which are arisen from the semi-invariance of M. Firstly, a basic classification theorem is obtained for M in terms of its mean curvature vector. Later, it is demonstrated that M is a totally geodesic submanifold if the dimension of the anti-invariant distribution $D^{\perp}$ is equal to the codimension of M. Finally, it is proven that M is an extrinsic sphere if the dimension of the invariant distribution D is greater than or equal to 2.

***K*eywords** Metallic Riemannian manifold · Semi-invariant submanifold · Totally umbilicality

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QUATERNIONS ON APPARENT MOVEMENT OF THE SUN ACCORDING TO VENUS

Deniz GUCLER

*A*nkara University, Department of Mathematics, Ankara, Türkiye

deniz.gucler@yahoo.com

ABSTRACT

The purpose of this article is to express the apparent movement of the Sun according to the planet Venus by using quaternions as a rotation operator and to compare the curve formed according to this planet with the curve of the apparent movement of the Sun according to Earth. In order to achieve this daily and yearly apparent movement of the Sun according to the planet Venus, how quaternions work as rotation operators, and the study done previously which shows the apparent movement of the Sun according to Earth by using quaternions has been examined. A quaternion that will be used as a rotation operator has been determined for each daily and yearly apparent movement of the Sun according to planet Venus. Afterward, these two rotational operators have been applied to the vector that is found between the planet Venus, which is accepted that it is found on point (0,0,0) of the coordinate system, and the accepted starting point of the daily and yearly apparent movement of the Sun according to the planet Venus, which it is accepted that it is found on point (1,0,0) of the coordinate system. As a result, a curve on a sphere is obtained and commentary on this curve is made. The importance of this work is that it shows the ease that the use of quaternions brings to the effort of showing both the daily and the yearly movements in the same curve. By managing to express both these movements in the same curve, the opportunity to better perceive the climatic changes between planets is given.

***K*eywords** Quaternions, apparent movement of the Sun, rotational motion.

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A New Hybrid Family of Conjugate Gradient Methods using Weak Wolfe Inexact Line Search

Amina Hallal[1], Mohammed Belloufi[2]

[1]*D*epartment of Computer Science and Mathematics, Laboratory Informatics and Mathematics (LiM) , Mohamed Cherif Messaadia University
[2]*D*epartment of Computer Science and Mathematics, Laboratory Informatics and Mathematics (LiM) , Mohamed Cherif Messaadia University

Corresponding Author's E-mail: a.hallal@univ-soukahras.dz

Abstract

The conjugate gradient method (CGM) is a well-known method for solving unconstrained problems. A new hybrid conjugate gradient method is created in this article using a novel combination of algorithms. The search direction sets the required descent condition for each iteration. Global convergence for common assumptions is achieved when the step size is calculated using the weak Wolfe inexact line search conditions. Additionally, we display numerical results for various benchmark test functions are used to prove the effectiveness of the new method and the new algorithm is competitive and improved than the other methods for big dimensions

***K*eywords** · ·

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When a 1-form is a magnetic map?

Bouazza KACIMI [1]

[1] University of Mascara, Department of Mathematics, 29000 Mascara, Algeria

Corresponding Author's E-mail: bouazza.kacimi@univ-mascara.dz

Abstract

Let (M, g) and (N, h) be two Riemannian manifolds of dimensions m and n respectively, and let $\varphi : (M, g) \to (N, h)$ a smooth map between them. The map φ is said to be harmonic if it is a critical point of the energy functional defined by: $E(\varphi) = \frac{1}{2} \int_D |d\varphi|^2 v_g$, for any compact domain D of M, where v_g is the volume element of (M, g). The energy functional $E(\varphi)$ has been widely studied by several researchers see for example [1, 3]. Now, let ξ be a global divergence free vector field on M and ω be a 1-form on N. The map φ is said to be magnetic if it is a critical point of the Landau Hall functional defined by: $LH(\varphi) = E(\varphi) + \int_D \omega(d\varphi(\xi))v_g$, or equivalently φ is a magnetic map with respect to ξ and ω if and only if it satisfies the Lorentz equation, that is $\tau(\varphi) = \phi(d\varphi(\xi))$, where $\tau(\varphi) = Tr_g(\nabla d\varphi)$ is the tension field of φ. The endomorphism ϕ, called the Lorentz force associated to the potential 1-form ω, is defined by $g(\phi(X), Y) = d\omega(X, Y)$, for all $X, Y \in \mathfrak{X}(M)$. The Landau Hall functional has been studied in [5] and also in [4]. Denote by g_S the Sasaki metric on the tangent bundle TM see [6]. Any $X \in \mathfrak{X}(M)$ determines a smooth map from (M, g) to (TM, g_S). In [4], Inoguchi and Munteanu established a characterization theorem for a vector field X viewed as a map from a Riemannian manifold (M, g) to its tangent bundle TM equipped with the Sasaki metric g_S to be magnetic map, moreover they proved that the characteristic vector field ξ on a Sasakian space form is magnetic map. In this work, we construct an almost complex structure J^S on the cotangent bundle T^*M and we prove that J^S is an almost Kahlerian structure on the cotangent bundle T^*M equipped with the Sasaki metric g^S, which allows us to provide a characterization theorem for a 1-form viewed as a map from a Riemannian manifold (M, g) to its cotangent bundle T^*M equipped with the Sasaki metric g^S to be magnetic map, moreover we prove that the 1-form η dual to ξ on a Sasakian space form is magnetic map.

Keywords Cotangent bundle · Sasaki metric · Magnetic map

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ON THE CLASSIFICATIONS OF NILSOLITON DERIVATIONS BY SYMBOLIC COMPUTATION

Hulya KADIOGLU[1]

[1] Yildiz Technical University, Türkiye

Corresponding Author's E-mail: hkadio@yildiz.edu.tr

ABSTRACT

A Riemannian metric is an important tool for the geometric structure of a given manifold. Particularly on Lie groups, it is possible to define several different Riemannian metrics. Considering any Riemannian metrics, Einstein metrics are the most preferable metric, as the Ricci tensor complies the Einstein metric. But, it is not possible to define Einstein metrics on non-abelian nilpotent Lie algebras. Therefore the notion of nilsoliton metrics became an alternative option with a weaker condition on a left invariant metric on a nilpotent Lie group G.

A nilsoliton is a Nilpotent Lie algebra with a Riemannian metric with the aforementioned weaker condition. Nilsolitons are an important topic in mathematics for several reasons. First, nilsoliton metric Lie algebras are unique up to isometry and scaling. A nilpotent Lie algebra is an Einstein nilradical if and only if it admits a nilsoliton metric. Therefore classification of nilsoliton metrics on a nilpotent Lie algebra is equivalent to the same of Einstein nilradicals. On the other hand, an Einstein solvmanifold can completely be determined by the Lie algebra with the Lie bracket of the solvmanifold with itself .Therefore the study of solvmanifolds are actually the study of nilsolitons.

Recently symbolic computation methods have been used for Lie algebras. By the formulation of algorithms, one can easily compute exact solutions of symbolic mathematical problems with the help of computer algebra programming languages. By this way, the computations can be done more productively and accurately than by hand. To use symbolic computation, we represent a Lie algebra by using table of its structure constants with a fixed basis explicitly by given multiplication table, consisting of structure constants.

In this study, we present a computational procedure for the classifications of all possible eigenvalues of simple nilsoliton derivations in dimension 9. We particularly classify the derivations of nilsolitons with non-singular Gram matrix. The computational procedure was implemented using Matlab R2022b with symbolic computation package on Intel(R)Core(TM) i3-5015U CPU at 2.10 GHz processor and 4 GB of RAM.

***K*eywords** Nilsoliton Metrics Nilradical Solvable Lie Algebra

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ISOTROPIC TRANSVERSAL LIGHTLIKE SUBMERSIONS

Esra Karataş[1], Cumali Yıldırım[2]

[1,2]Department of Mathematics, Faculty of Science and Literature, Inonu University, Malatya, Türkiye

Corresponding Author's E-mail: esrakrts4422@gmail.com

ABSTRACT

We're going to introduce an isotropic transversal lightlike submersion called $f : M_1 \to M_2$ in this paper. Additionally using the fundamental tensor fields T and A, we will obtain several equations for isotropic transversal lightlike submersions and reach some significant conclusions.

Keywords Isotropic transversal submersion · Riemannian submersion · lightlike submersion.

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UNIQUENESS OF POSITIVE SOLUTIONS OF A FRACTIONAL INTEGRO-DIFFERENTIAL EQUATION BY SCHAUDER FIXED POINT THEOREM

Chahra Kechar [1], Abdelouaheb Ardjouni [2]

[1]*D*epartment of Mathematics and Informatics, Souk Ahras University, Souk Ahras, Algeria
[2]*D*epartment of Mathematics and Informatics, Souk Ahras University, Souk Ahras, Algeria

E-mail: chahra95kechar@gmail.com

ABSTRACT

We study the existence and uniqueness of positive solutions of a fractional integro-differential equation with integral boundary conditions. We convert the given fractional integro-differential equation into an equivalent integral equation. Then we construct appropriate mappings and employ the Schauder fixed point theorem and the method of upper and lower solutions to show the existence of a positive solution. We also use the Banach fixed point theorem to show the existence of a unique positive solution. Finally, an example is given to illustrate our results.

***K*eywords** Fractional integro-differential equations; Positive solutions; Upper and lower solutions; Fixed point theorem.

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TWO SERVER INTERDEPENDENT QUEUEING MODEL WITH BULK SERVICE

Shavej Ali Siddiqui[1]

[1]Khwaja Moinuddin Chishti Language University, India

Corresponding Author's E-mail: shavej.amu@gmail.com

ABSTRACT

We study two service facilities are independent of each other and the arrival and service processes are interdependent. This sort of situations are more common in marshalling yard's with two engines, elevator process with two lifts etc. In both the models we first develop the difference-differential equations and solve them through generating function techniques.

Keywords The dependence parameter · Mean arrival rate · Mean dependence rate

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BICONSERVATIVE PNMCV SURFACES IN MINKOWSKI SPACES

Rüya Yeğin Şen

Istanbul Medeniyet University, Faculty of Engineering and Natural Sciences, Department of Mathematics, 34700 Üsküdar, İstanbul, Türkiye

Corresponding Author's E-mail: ruya.yegin@medeniyet.edu.tr

ABSTRACT

Biharmonic maps are a natural generalization of harmonic maps. A map ψ is called *biharmonic* if it is a critical point of the bi-energy functional E_2. G.Y. Jiang studied the first and second variation formulas of E_2 for which critical points are called biharmonic maps, (see [4]). The Euler-Lagrange equation associated with this bi-energy functional is

$$\tau_2(\psi) = 0, \tag{5}$$

where τ_2 is the bi-tension field.

The well known Chen's conjecture on biharmonic submanifolds in Euclidean spaces states that every biharmonic submanifold in a Euclidean space is a minimal one. In the last decades, biharmonic submanifolds have become a popular subject of research with many significant progresses made by geometers around the world. The conjecture has been verified in the lots of papers by considering additional geometric properties. However, Chen's conjecture is still open.

On the other hand, a mapping $\psi : (\Omega, g) \to (N, \tilde{g})$ satisfying the condition

$$\langle \tau_2(\psi), d\psi \rangle = 0, \tag{6}$$

that is weaker than (5) is said to be biconservative. When ψ is an isometric immersion, the equation (6) turns into

$$\tau_2(\psi)^T = 0,$$

where $\tau_2(\psi)^T$ denotes the tangential part of $\tau_2(\psi)$. In this case, Ω is said to be a biconservative submanifold of N.

To understand geometry of biharmonic submanifolds, biconservative submanifolds also have been studied in many papers so far.

The study of submanifolds with parallel normalized mean curvature vector in Euclidean spaces was initiated at the beginning of the 1980s (see [1]). A submanifold which has parallel normalized mean curvature vector field (PNMCV), if it has

nowhere zero mean curvature and the unit normal vector field in the direction of the mean curvature vector field is parallel in the normal bundle, i.e., $D(H/f) = 0$ where $f = |H|$ and D are the mean curvature and the normal connection, respectively.

In this study, we investigate biconservative surfaces with parallel normalized mean curvature vector field (PNMCV) in the Minkowski space $\mathbb{E}^m_1$. We obtain canonical forms of the shape operators of such submanifolds. We obtain the local parametrization of these surfaces in $\mathbb{E}^m_1$.

Keywords Biconservative surfaces · Parallel normalized mean curvature vector field · Biharmonic submanifolds

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Geometric Modeling in Medicine

Ferhat TAŞ[1]

[1]*Department of Mathematics, Faculty of Science, İstanbul University, Türkiye*

Corresponding Author's E-mail: tasf@istanbul.edu.tr

Abstract

Geometric modeling has become increasingly important in the field of medical treatment of some diseases. In this oral presentation, we will discuss two subjects in medicine: the geometric modeling of the aortic root and the mathematical-geometric modeling of deformation of the cornea resulting in keratoconus disease.

The aortic root, which is the portion of the aorta that connects to the heart, is prone to a variety of degenerative diseases that can lead to significant morbidity and mortality. One of the most common treatments for aortic root disease is surgical replacement with a prosthetic graft. However, the success of this surgery is dependent on the precise fit of the graft to the patient's aortic root.

To ensure a precise fit, surgeons often rely on CT and MRI images to create 3D models of the patient's aortic root. These models can then be used to design and manufacture customized prosthetic grafts using 3D printing technology. In this presentation, we will discuss the use of rational Bézier curves, a type of mathematical curve used in geometric modeling, to design and manufacture these prosthetic grafts.

In addition to their use in cardiovascular surgery, geometric modeling techniques have also been applied in the treatment of keratoconus disease, a condition in which the cornea becomes thin and cone-shaped, leading to vision impairment. In the treatment of keratoconus, geometric modeling is used to understand the biomechanics of the cornea and the deformations it undergoes during the disease process. This understanding can then be used to design and manufacture customized contact lenses that can help restore vision in patients with keratoconus.

Overall, the use of geometric modeling in medicine has the potential to revolutionize the way we treat a variety of conditions, including cardiovascular disease and keratoconus. By using mathematical techniques to create precise 3D models of the body's structures and deformations, we can design and manufacture customized medical devices that fit perfectly, leading to better patient outcomes.

***K*eywords** Aortic root · Cardiovascular surgery · Replacement · CT and MRI images · Geometric modeling · Rational Bézier Curves · 3D printing · Biomechanics of Cornea · Deformations

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SEMI-INVARIANT $\xi^{\perp}-$RIEMANNIAN SUBMERSIONS ADMITTING RICCI SOLITON

Ramazan Sarı[1], İnan Ünal[2]

[1]Gümüshacıköy Hasan Duman Vocational School, Amasya University, Amasya, Türkiye
[2]Department of Computer Engineering, University of Munzur, Tunceli, Türkiye

Corresponding Author's E-mail: ramazan.sari@amasya.edu.tr

ABSTRACT

In this paper, we work Ricci solitons on semi invariant Riemannian submersions from a contact manifold. We investigate the foliations of the submersions as a Ricci soliton. Moreover, we examine Einstein conditions.

Keywords Riemannian submersion; semi-invariant Riemannian submersion; Ricci soliton; space forms.

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RICCI SOLITONS WITH CONCIRCULAR AND CONFORMAL KILLING POTENTIAL VECTOR FIELDS IN COMPLEX SASAKIAN MANIFOLDS

Aysel TURGUT VANLI

Gazi University, Fac. of Sci., Dept. of Math. 06500 Ankara /TÜRKİYE
Corresponding Author's E-mail: avanli@gazi.edu.tr

ABSTRACT

Real Sasakian manifolds are well known and there are many articles in the literature on this subject. Complex Sasakian manifolds are complex analogue of Sasakian manifolds. But there are very few articles in the literature on complex Sasakian manifolds. Foreman was the first to work on complex Sasakian manifolds in 2000. Ban-Yen Chen first introduced and classified Ricci solitons with congruent and concircular vector fields on a Riemannian manifold in 2015. In this paper, Ricci solitons (g, ν, λ) on a complex Sasakian manifold has concurrent, concircular and conformal killing potential vector fields, respectively ν are studied. It is also shown that a Ricci soliton in complex Sasakian manifolds satisfying $\rho(U, X)R = 0$ and $\rho(V, X)R = 0$ is always expanding.

Keywords Complex Sasakian · Ricci soliton · concurrent · concircular · conformal killing

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Participant List

Sudhakar Kumar CHAUBEY, University of Tech. and Appl. Sci., Shinas (Oman), Invited Speaker

Ryszard DESZCZ, Wrocław University of Environmental and Life Sciences (Poland), Invited Speaker

Sorin DRAGOMIR, Università degli Studi della Basilicata (Italy), Invited Speaker

Dorel FETCU, Asachi Technical University of Iasi (Romania), Invited Speaker

Eduardo GARCIA-RIO, University of Santiago de Compostela (Spain), Invited Speaker

Mohammad Nazrul Islam KHAN, Qassim University (Saudi Arabia), Invited Speaker

Adela MIHAI, Technical University of Civil Engineering Bucharest (Romania), Invited Speaker

Sema SALUR, University of Rochester (USA), Invited Speaker

Venkatesha VENKATESHA, Karnata University (India), Invited Speaker

Chris WOOD, University of York (UK), Invited Speaker

Şeyma AFŞAR, Gazi University (Türkiye), Listener

Alev KELLECİ AKBAY, Iskenderun Technical University (Türkiye), Listener

Ömer AKGÜLLER, Muğla Sıtkı Koçman University (Türkiye), Listener

Çisem Güneş AKTAŞ, Abdullah Gül University (Türkiye), Oral Presentation

Gamze ALKAYA, Gazi University (Türkiye), Listener

Anıl ALTINKAYA, Gazi University (Türkiye), Listener

Rümeysa Sacide ALTINKAYA, Gazi University (Türkiye), Listener

Gül ASAN, Ankara University (Türkiye), Listener

Muhittin Evren AYDIN, Firat University (Türkiye), Listener

Ajit BARMAN, Ramthakur College (India), Listener

Duygu Gülşah BAŞARAN, Gazi University (Türkiye), Listener

Büşra BATARAY, Çanakkale Onsekiz Mart University (Türkiye), Listener

Selahattin BEYENDİ, Inonu University (Türkiye), Oral Presentation

Onur BİLGE, Gazi University (Türkiye), Listener

Neslihan BİRİCİK, Gazi University (Türkiye), Listener

Adara M. BLAGA, West University of Timisoara (Romania), Listener

Satyanarayana BORA, University of Technology and Applied Sciences-Shinas (Oman), Listener

Abir BOUNAAMA, 20 August 1955 (Algeria), Oral Presentation

Hülya GÜN BOZOK, Osmaniye Korkut Ata University (Türkiye), Oral Presentation

Saniye CAN, Çanakkale Onsekiz Mart University (Türkiye), Listener

Marcos CAVALCANTE, Federal University of Alagoas (Brazil), Listener

OMAR CHOUCHA, Université Kasdi Merbah Ouargla (Algeria), Oral Presentation

Majid Ali CHOUDHARY, Maulana Azad National Urdu University (India), Oral Presentation

Duygu ÇAĞLAR, Yildiz Technical University (Türkiye), Listener

Mert ÇARBOĞA, Ankara University (Türkiye), Oral Presentation

Kübra ÇETİNBERK, Yildiz Technical University (Türkiye), Listener

Sezin ÇİT, Gazi University (Türkiye), Listener

Mehriban DALIEYUBOVA, Gazi University (Türkiye), Listener

Elif Ece DEMİR, Gazi University (Türkiye), Listener

Yusuf DOĞRU, Hava Eğitim Komutanlığı (Türkiye), Oral Presentation

Onur Talip DOSTUOĞLU, Osmaniye Korkut Ata University (Türkiye), Oral Presentation

Gökçe EFEOĞLU, Gazi University (Türkiye), Listener

Ömer EREN, Gazi University (Türkiye), Listener

İrem KÜPELİ ERKEN, Bursa Technical University (Türkiye), Listener

Małgorzata GŁOGOWSKA, Wrocław University of Environmental and Life Sciences (Poland), Listener

Mustafa GÖK, Sivas Cumhuriyet University (Türkiye), Oral Presentation

Deniz GÜÇLER, Ankara University (Türkiye), Oral Presentation

Sinem GÜLER, Istanbul Sabahattin Zaim University (Türkiye), Listener

Mehmet GÜMÜŞ, Çanakkale Onsekiz Mart University (Türkiye), Listener

Nurten GÜRSES, Yildiz Technical University (Türkiye), Listener

Gizem GÜZELKARDEŞLER, Manisa Celal Bayar University (Türkiye), Listener

Amina HALLAL, Mohamed Cherif Messaadia University (Algeria), Oral Presentation

Bouazza KACIMI, University of Mascara (Algeria), Oral Presentation

Hülya KADIOĞLU, Yildiz Technical University (Türkiye), Oral Presentation

Sıddıka ÖZKALDI KARAKUŞ, Bilecik Şeyh Edebali University (Türkiye), Listener

Esra KARATAŞ, Inonu University (Türkiye), Oral Presentation

Chahra KECHAR, University of Souk Ahras (Algeria), Oral Presentation

Dilek KEKEÇ, Yildiz Technical University (Türkiye), Listener

Sumeet KUMAR, B. R. Ambedkar Bihar University (India), Listener

Yasin KÜÇÜKARIKAN, Yozgat Bozok University (Türkiye), Listener

Pavan KUMAR MAURYA, SRI RAMSWAROOP MEMORIAL UNIVERSITY (India), Listener

Mücahit MERAL, Yozgat Bozok University (Türkiye), Listener

Azouzi MOUNIRA, Echahid hamma lakhder university of Eloued (Algeria), Oral Presentation

Ameth NDIAYE, Université Cheikh Anta Diop (Sénégal), Listener

Nurdan OĞRAŞ, Erciyes University (Türkiye), Listener

Turgut ÖNDER, Middle East Technical University (Türkiye), Listener

Haşim ÖZEN, Anadolu University (Türkiye), Listener

Cihan ÖZGÜR, Izmir Democracy University (Türkiye), Listener

Mustafa ÖZKAN, Gazi University (Türkiye), Listener

Mustafa ÖZKAN, Bursa Technical University (Türkiye), Listener

Güler Başak ÖZNUR, Gazi University (Türkiye), Listener

Burçin SALTIK, Erciyes University (Türkiye), Listener

Ramazan SARI, Amasya University (Türkiye), Oral Presentation

Furkan SEÇGİN, Gazi University (Türkiye), Listener

Bilge Zehra SERGİ, Gazi University (Türkiye), Listener

Emre SEVGİ, Gazi University (Türkiye), Listener

Shavej Ali SIDDIQUI, Khwaja Moinuddin Chishti Language University (India), Oral Presentation

Gülşah AYDIN ŞEKERCİ, Süleyman Demirel University (Türkiye), Listener

Rüya YEĞİN ŞEN, Istanbul Medeniyet University (Türkiye), Oral Presentation

Gülsüm Yeliz ŞENTÜRK, Istanbul Gelisim University (Türkiye), Listener

Batuhan TARCAN, Gazi University (Türkiye), Listener

Ferhat TAŞ, Istanbul University (Türkiye), Oral Presentation

Pelin TEKİN, Trakya University (Türkiye), Listener

Mustafa Deniz TÜRKOĞLU, Haliç University (Türkiye), Listener

Esra Esin TÜTÜNCÜ, (Türkiye), Listener

Selin ULUSAY, Bilkent University (Türkiye), Listener

İnan ÜNAL, Munzur University (Türkiye), Oral Presentation

Aysel TURGUT VANLI, Gazi University (Türkiye), Oral Presentation

Yusuf YAYLI, Ankara University (Türkiye), Oral Presentation

Serdal YAZICI, Gazi University (Türkiye), Listener

Özlem YETÜT, Trakya University (Türkiye), Listener

Neslihan YILDIRIM, Gazi University (Türkiye), Listener

GAZİ UNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS

INTERNATIONAL GEOMETRY SYMPOSIUM

In Memory of
Prof. Erdoğan ESİN