Lecture 2

Bergman metric, Tangentially C-R Foliations, Graham-Lee Connection, Boundary Values of Bergman-Harmonic Maps and Yang-Mills fields on Strictly Pseudoconvex Domains[1]

Sorin Dragomir[2]

The Bergman kernel $K(z,\,\zeta)$ of a (bounded) domain $\Omega \subset \mathbb{C}^n$, the Bergman metric $(g_\Omega)_{j\overline{k}} = \partial^2 \log K(z,\,z)/\partial z^j\,\partial \overline{z}^k$ (cf. [Hel]). Neumann's operator for the $\overline{\partial}$ operator on a smoothly bounded strictly pseudoconvex domain $\Omega \subset \mathbb{C}^n$, $n \geq 2$ (cf. [Koh]), and applications to the regularity up to the boundary of the Bergman kernel of Ω (cf. [Ker]). Fefferman's asymptotic formula for the Bergman kernel (cf. [Fef]) and its many consequences i.e. the holomorphic sectional curvature of $(\Omega\,,\,g_\Omega)$ approaches towards the boundary $\partial\Omega$ the constant holomorphic sectional curvature of the unit ball $(\mathbb{B}^n\,,\,g_{\mathbb{B}^n})$ (cf. [Kle] and [Bar]), boundary values $f : \partial\Omega \to \partial\Omega$ of symplectomorphisms $F : \Omega \to \Omega$ (that extend smoothly at the boundary) are contact transformations (cf. [KoRe], [BaDr]). One sided neighborhoods $(V,\,\mathscr{F})$ of the boundary $\partial\Omega$ of a smoothly bounded, strictly pseudoconvex domain $\Omega \subset \mathbb{C}^n$, foliated by level sets of the defining function $\varphi(z) = -K(z,\,z)^{-1/(n+1)}$, the transverse curvature $r = 2\,(\partial\,\overline{\partial}\,\varphi)(\xi,\,\overline{\xi})$ of $\mathscr{F}$ is smooth up to the boundary (cf. [LeMe]). The Graham-Lee connection ∇ of $(V,\,\mathscr{F})$ (cf. [GrLe]). The Dirichlet problem for the Bergman Laplacian Δ_Ω of $(\Omega\,,\,g_B)$, i.e. $\Delta_\Omega u = 0$ in Ω and $u = f$ on $\partial\Omega$, and the C^∞ regularity up to the boundary problem, geometric methods for the determination of compatibility conditions $\mathscr{C}(f) = 0$ along $\partial\Omega$ [that are necessary for $u \in C^\infty\big(\overline{\Omega}\big)$] (cf. [GrLe]). Graham-Lee's geometric approach to the C^∞ regularity up to the boundary problem for Bergman-harmonic maps $\Phi : \Omega \to N$ of a strictly pseudoconvex domain $(\Omega,\,g_B)$ into a Riemannian manifold N

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`https://geometrysummerschool.gazi.edu.tr/`

[2]Università degli Studi della Basilicata, Dipartimento di Scienze Agrarie, Forestali, Alimentari e Ambientali, Potenza, Italy, e-mail: `sorin.dragomir@unibas.it`
King Khalid University, Department of Mathematics, Abha, Saudi Arabia, e-mail: `sorin.dragomir955@gmail.com`

and for Yang-Mills fields $D \in \mathscr{C}(\mathbb{E})$ on a vector bundle $\mathbb{E} \to \overline{\Omega}$ (cf. [DrKa], [BDU]).

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